> with(plots)

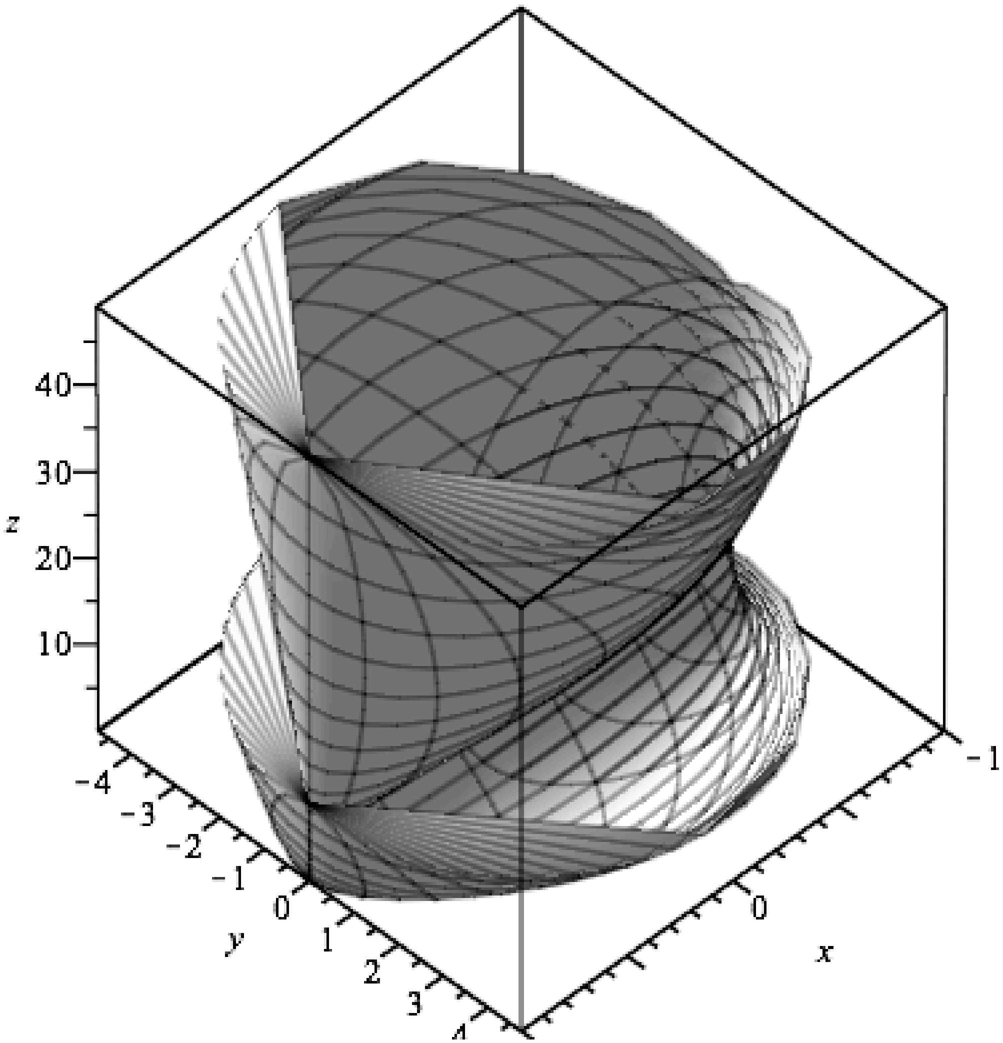
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot] (1

Ekstraoppgave 11.8.1.

a)

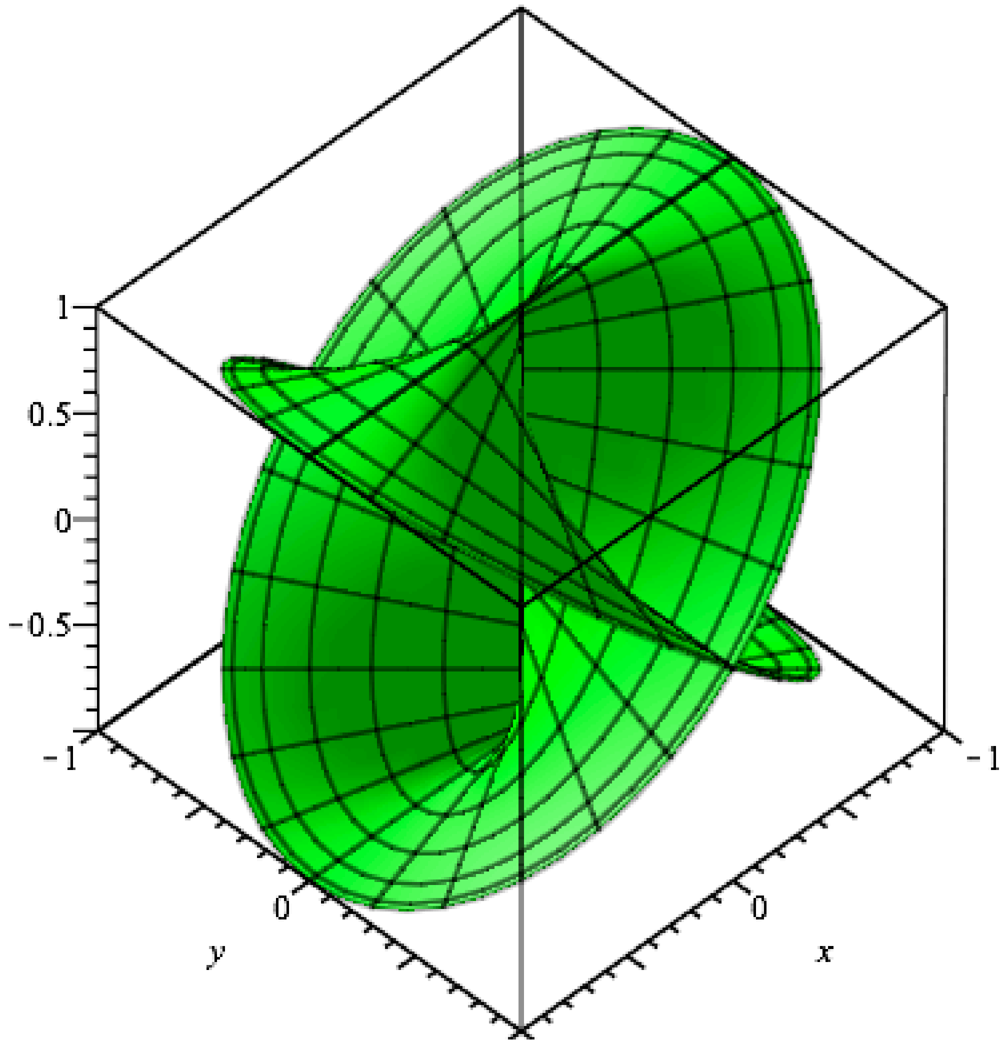
Vi bruker akkurat samme kommandoer for å tegne parametriserte flater som for å tegne flatene i kapittel 11.7.
Faktisk, ser du nøyere på mapleoppgavene i 11.7, ser du at vi parametriserte flatene med hensyn på to av de tre fri variable.

> $plot3d\left(\left[\cos(v), u\cdot\sin(v), (u-2)^2\right], u=-5..5, v=-\mathrm{sqrt}\left(5^2-u^2\right)..\mathrm{sqrt}\left(5^2-u^2\right), color=grey, axes=boxed, labels=[x,y,z]\right)$



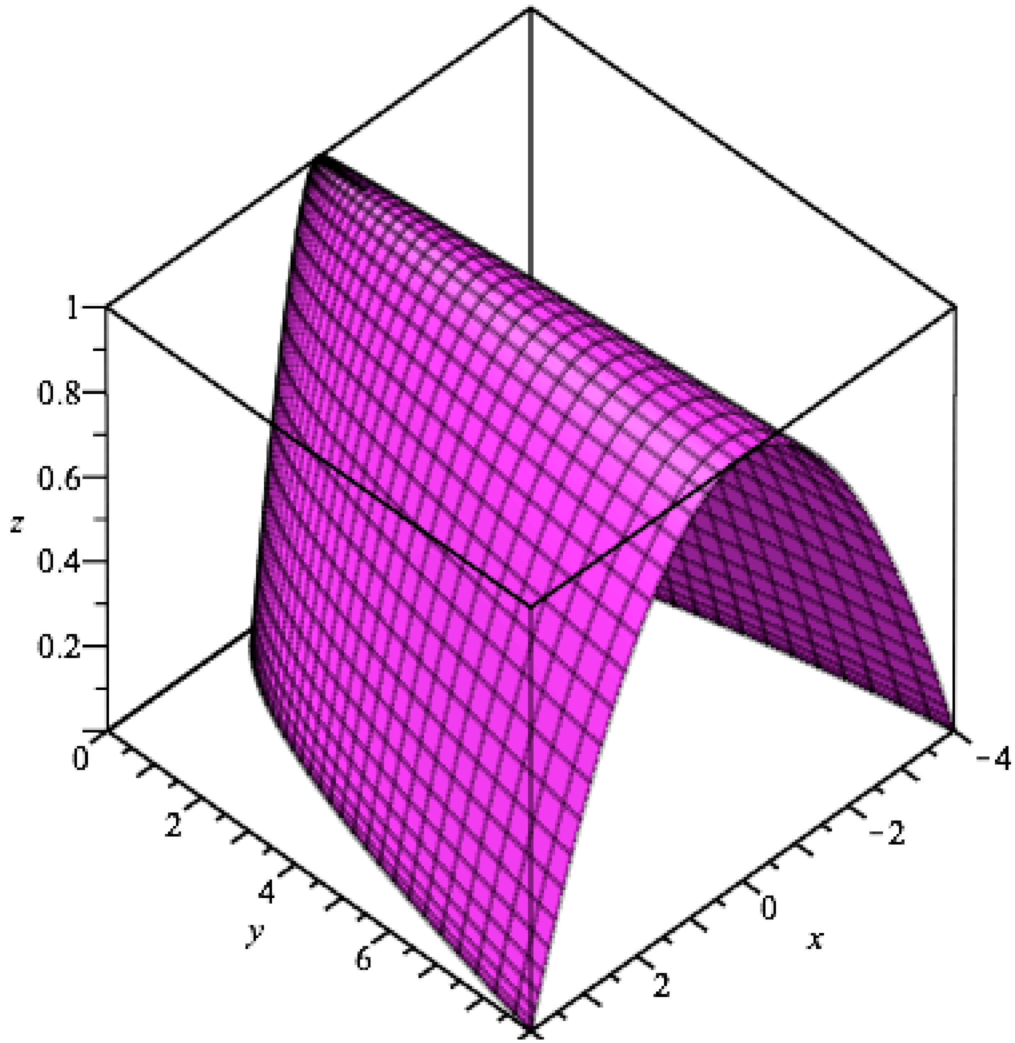
b)

> $plot3d([\sin(u)\cdot\cos(v), \sin(u)\cdot\sin(v), \cos(v)], u = 0..2\cdot\text{Pi}, v = 0..2\cdot\text{Pi}, color = green, axes = boxed, labels = [x, y, z])$



c)

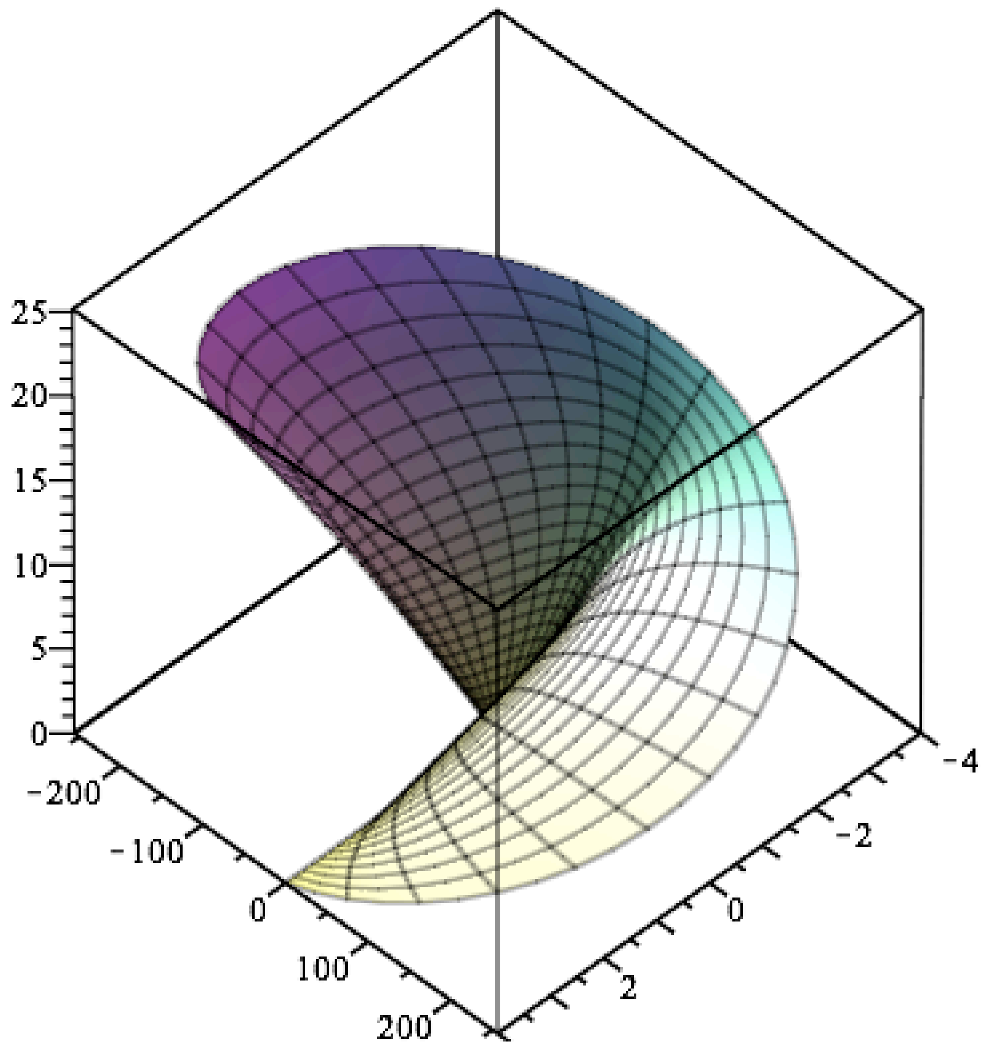
> $plot3d\left(\left[r\cdot\cos(t),\ (r-1)^2,\ \sin(t)^2\right], r = 0..4, t = 0..Pi, color = magenta, axes = boxed, numpoints = 50000, labels = [x, y, z]\right)$



Ekstraoppgave 11.8.2.

a)

> $plot3d\left(\left[u\cdot\cos(v),\ u^4\cdot\sin(v),\ u\cdot v\right],\ u = 0..4,\ v = 0..2\cdot \mathrm{Pi},\ axes = boxed\right)$



Aha. Slik ser altså flaten ut. Men vi skal ha den plottet sammen med tangentplanet i punktet $\boldsymbol{r}\left(2,\ \frac{\pi}{4}\right)$.

For ikke å få så mange streker på bildet, fjerner vi også rutenettet vi fikk på flaten, og ber om en ensfarget figur.

> $P1 := plot3d\left(\left[u\cdot\cos(v),\ u^4\cdot\sin(v),\ u\cdot v\right],\ u=0..4,\ v=0..2\cdot\mathrm{Pi},\ color=green\right)$

$$P1 := PLOT3D(\ldots) \quad (2$$

For å finne likningen til tangentplanet, trenger vi en normalvektor til flaten i punktet.

Setter vi $T = \langle u\cdot\cos(v),\ u^4\cdot\sin(v),\ u\cdot v\rangle$, er det enkelt å finne de partiellderiverte til T med hensyn på henholdsvis u og v, la oss kalle dem Tu og Tv.

Da er kryssproduktet $Tu\left(2, \frac{\pi}{4}\right) \times Tv\left(2, \frac{\pi}{4}\right)$ en slik normalvektor.

Men før vi setter igang med vektorregning, importerer vi Maples kommandoer for slike beregninger:

> $with(VectorCalculus)$

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series] (3

> $T := (u, v) \rightarrow \langle u\cdot\cos(v),\ u^4\cdot\sin(v),\ u\cdot v\rangle$

$$T := (u, v) \rightarrow VectorCalculus{:-}{<,>}\left(u\cos(v), u^4\sin(v), u\, v\right) \quad (4$$

> $\mathrm{Tu} := (\mathrm{u}, \mathrm{v}) \rightarrow diff(T(u, v), u)$

$$Tu := (u, v) \rightarrow \frac{\partial}{\partial u} T(u, v) \quad (5$$

> $\mathrm{Tv} := (\mathrm{u}, \mathrm{v}) \rightarrow diff(T(u, v), v)$

$$Tv := (u, v) \rightarrow \frac{\partial}{\partial v} T(u, v) \quad (6$$

> $\mathrm{CP} := CrossProduct(\mathrm{Tu}(\mathrm{u}, \mathrm{v}), Tv(\mathrm{u}, \mathrm{v}))$

$$CP := \left(4\, u^4 \sin(v) - v\, u^4 \cos(v)\right) e_x + \left(-u\cos(v) - v\, u\sin(v)\right) e_y + \left(\cos(v)^2 u^4 + 4\, u^4 \sin(v)^2\right) e_z \quad (7$$

Vi trenger å sette at

$(x, y, z) = \boldsymbol{r}\left(2, \frac{\pi}{2}\right)$ både inn i dette kryssproduktet og i selve $T(u, v)$ (for å vite tangeringspunktet).

> $CPs := subs\left(u = 2, v = \frac{\mathrm{Pi}}{2}, CP\right)$

$$CPs := 64e_x - \pi e_y + 64e_z \tag{8}$$

> $Ts := subs\left(u = 2, v = \frac{\mathrm{Pi}}{2}, \mathrm{T}(\mathrm{u}, \mathrm{v})\right)$

$$Ts := 2\cos\left(\frac{1}{2}\pi\right)e_x + 16\sin\left(\frac{1}{2}\pi\right)e_y + (\pi)e_z \tag{9}$$

> $simplify(Ts)$

$$16e_y + (\pi)e_z \tag{10}$$

Nå er det lett å sette opp likningen for tangentplanet:

$64(x - 0) - \pi(y - 16) + 64(z - \pi) = 0.$

For å plotte dette planet løser vi først likningen for planet med hensyn på for eksempel z :

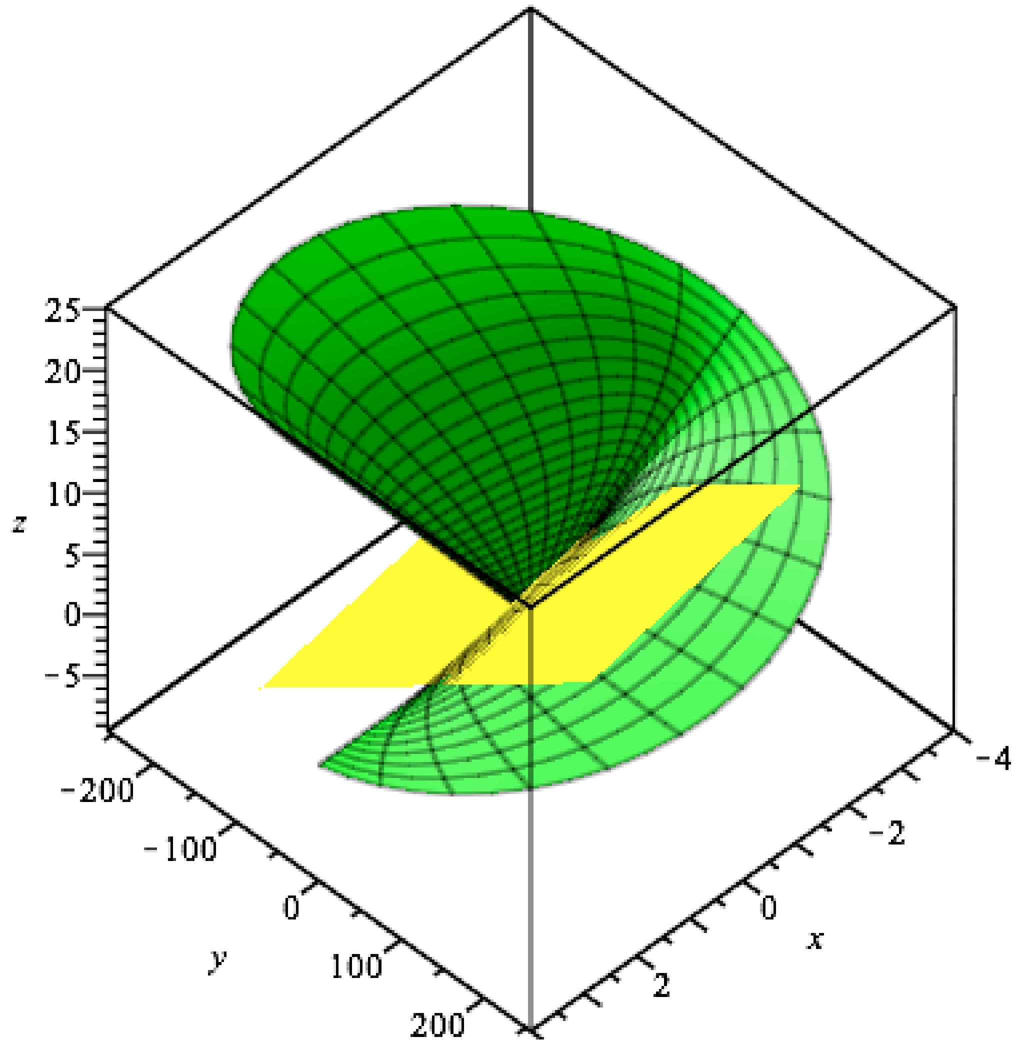
> $\mathrm{solve}(64\,x - \pi\,(y - 16) + 64\,z - 64\,\pi = 0, z)$

$$\frac{1}{64}\pi y + \frac{3}{4}\pi - x \tag{11}$$

> $P2 := plot3d\left(\left[x, y, -x + \frac{1}{64}\pi y + \frac{3}{4}\pi\right], x = -2..2, y = -200..200,\ style = surface, color = yellow\right)$

$$P2 := PLOT3D(...) \tag{12}$$

> $display(P1, P2, axes = boxed, labels = [x, y, z])$

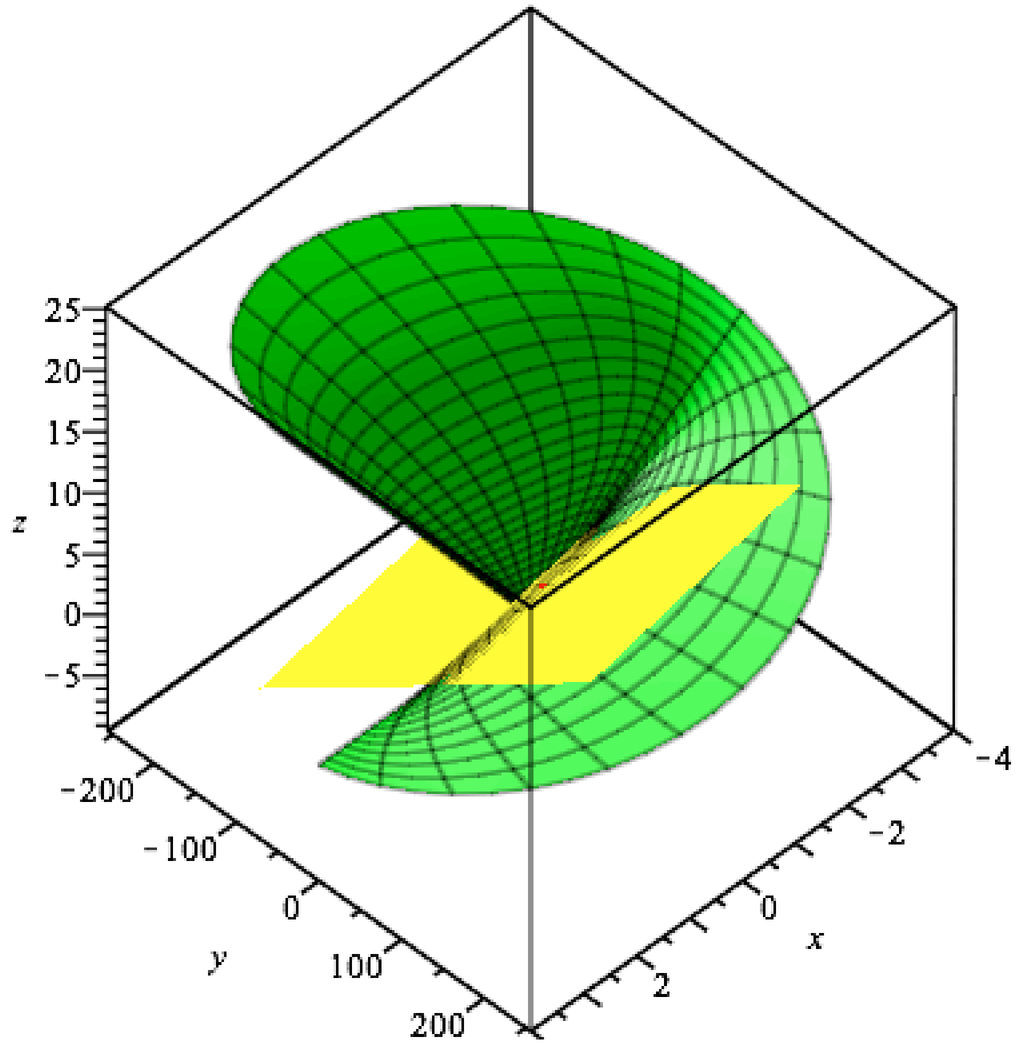


Bildet blir klarere om vi også plotter tangeringspunktet. Det kan vi gjøre med følgende kommando:

> *P3* := *pointplot3d*([0, 16, Pi], *color* = *red*)

$$P3 := PLOT3D(\ldots) \quad (13$$

> *display*(*P1*, *P2*, *P3*, *axes* = *boxed*, *labels* = [*x*, *y*, *z*])



Det røde tangeringspunktet er ganske lite, men du ser det på figuren hvis du kikker nøye!!!

>