Oppgave 10.6.10.

Kommandoen for taylorrekker for funksjoner av flere variable heter *mtaylor*.
Ved *taylor* måtte man be om en orden høyere enn den vi vil ha. Slik er det også for *mtaylor*.
Sentrum for rekken må skrives i en hakeparentes eller krølleparentes.

a)

> $mtaylor\left(\mathrm{sqrt}\left(x^3+y^2\right),\ \{x=2, y=3\}, 6\right)$

$$\sqrt{17}+\frac{6}{17}\sqrt{17}\,(x-2)+\frac{3}{17}\sqrt{17}\,(y-3)+\frac{33}{289}\sqrt{17}\,(x-2)^2-\frac{18}{289}\sqrt{17}\,(y-3)\,(x-2)+\frac{4}{289}\sqrt{17}\,(y-3)^2 -\frac{107}{9826}\sqrt{17}\,(x-2)^3+\frac{9}{4913}\sqrt{17}\,(y-3)\,(x-2)^2+\frac{30}{4913}\sqrt{17}\,(y-3)^2\,(x-2)-\frac{12}{4913}\sqrt{17}\,(y-3)^3 -\frac{447}{167042}\sqrt{17}\,(x-2)^4+\frac{1401}{167042}\sqrt{17}\,(y-3)\,(x-2)^3-\frac{501}{83521}\sqrt{17}\,(y-3)^2\,(x-2)^2+\frac{54}{83521}\sqrt{17}\,(y-3)^3\,(x-2)+\frac{28}{83521}\sqrt{17}\,(y-3)^4+\frac{6213}{2839714}\sqrt{17}\,(x-2)^5-\frac{9585}{2839714}\sqrt{17}\,(y-3)\,(x-2)^4 +\frac{581}{2839714}\sqrt{17}\,(y-3)^2\,(x-2)^3+\frac{2079}{1419857}\sqrt{17}\,(y-3)^3\,(x-2)^2-\frac{666}{1419857}\sqrt{17}\,(y-3)^4\,(x-2) -\frac{36}{1419857}\sqrt{17}\,(y-3)^5 \quad (1$$

Dette er det eksakte taylorpolynomet.
Er man fornøyd med tilnærmede koefisienter, med for eksempel 4 sikre sifre, får vi det ved å sette

> $evalf(\%, 4)$

$$-0.970+1.455\,x+0.7277\,y+0.4708\,(x-2.)^2-0.2568\,(y-3.)\,(x-2.)+0.05706\,(y-3.)^2-0.04490\,(x-2.)^3 +0.007553\,(y-3.)\,(x-2.)^2+0.02518\,(y-3.)^2\,(x-2.)-0.01007\,(y-3.)^3-0.01103\,(x-2.)^4+0.03458\,(y-3.)\,(x-2.)^3-0.02473\,(y-3.)^2\,(x-2.)^2+0.002666\,(y-3.)^3\,(x-2.)+0.001382\,(y-3.)^4+0.009021\,(x-2.)^5 \quad (2$$

$$- 0.01392\ (y - 3.)\ (x - 2.)^4 + 0.0008436\ (y - 3.)^2\ (x - 2.)^3 + 0.006036\ (y - 3.)^3\ (x - 2.)^2 - 0.001934\ (y - 3.)^4\ (x - 2.) - 0.0001045\ (y - 3.)^5$$

Det er et stort arbeid å kontrollere alle disse koeffisientene. Vi tar bare en stikkprøvekontroll. Koeffisienten foran $(y-3)(x-2)^4$ er for eksempel lik $-\frac{9585}{2839714}\sqrt{17}$ ifølge Maples utregning.

Formlene våre sier at den skal være lik $\frac{5}{5!}\cdot\frac{\delta^5 f}{\delta x^4 \delta y}$ evaluert i punktet $(2, 3)$. Vi prøver:

> $diff\left(\mathrm{sqrt}(x^3+y^2), x, x, x, x, y\right)$

$$\frac{8505}{16}\frac{x^8 y}{(x^3+y^2)^{9/2}} - \frac{1215}{2}\frac{x^5 y}{(x^3+y^2)^{7/2}} + \frac{135\, x^2 y}{(x^3+y^2)^{5/2}} \tag{3}$$

> $subs(x = 2, y = 3, \%)$

$$-\frac{115020}{1419857}\sqrt{17} \tag{4}$$

> $\frac{5\cdot\%}{5!}$

$$-\frac{9585}{2839714}\sqrt{17} \tag{5}$$

Det stemmer perfekt!

Vi kontrollerer koeffisienten $\frac{581}{2839714}\sqrt{17}$ foran leddet $(y-3)^2(x-2)^3$ på den samme måten:

> $diff\left(\mathrm{sqrt}(x^3+y^2), x, x, x, y, y\right)$

$$\frac{2835}{8}\frac{x^6 y^2}{(x^3+y^2)^{9/2}} - \frac{405}{8}\frac{x^6}{(x^3+y^2)^{7/2}} - \frac{405}{2}\frac{x^3 y^2}{(x^3+y^2)^{7/2}} + \frac{81}{2}\frac{x^3}{(x^3+y^2)^{5/2}} + \frac{9\, y^2}{(x^3+y^2)^{5/2}} - \frac{3}{(x^3+y^2)^{3/2}} \tag{6}$$

> $subs(x = 2, y = 3, \%)$

$$\frac{3486}{1419857}\sqrt{17} \tag{7}$$

> $\frac{10 \cdot \%}{5!}$

$$\frac{581}{2839714} \sqrt{17} \tag{8}$$

Bingo!

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