Oppgave 12.6.20.

a)

Vi importerer først vektoranalysekommandoene til Maple:

> $with(VectorCalculus)$

$[\&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, \mathrm{D}, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series]$ **(1**

Vi starter akkurat som for curlen til et vektorfelt, men det blir litt enklere etterhvert, for divergensen er bare en vanlig skalar funksjon

> $SetCoordinates('cartesian'[x, y, z])$

$$cartesian_{x, y, z}$$ **(2**

> $F := VectorField\left(\left\langle \sin\left(x^{\frac{3}{2}}\right)\cdot\cos(z),\ \cos\left(x^{\frac{3}{2}}\right)\cdot\cos(z),\ \ln(\tan(x\cdot y\cdot z))\right\rangle\right)$

$$F := \left(\sin\left(x^{3/2}\right)\cos(z)\right)\bar{e}_x + \left(\cos\left(x^{3/2}\right)\cos(z)\right)\bar{e}_y + \left(\ln(\tan(x\,y\,z))\right)\bar{e}_z$$ **(3**

> $g := (x, y, z) \rightarrow Divergence(F)$

$$g := (x, y, z) \rightarrow VectorCalculus{:-}Divergence(F)$$ **(4**

Vi valgte å lage en funksjon $g(x, y, z)$ som er lik divergensen. Ulempen er at vi da må skrive $g(x, y, z)$ for å se hvordan den ser ut:

> g(x, y, z)

$$\frac{3}{2}\cos\left(x^{3/2}\right)\sqrt{x}\,\cos(z)+\frac{\left(1+\tan(x\,y\,z)^2\right)x\,y}{\tan(x\,y\,z)} \tag{5}$$

> $subs\left(x=\pi^{\frac{2}{3}}, y=\pi^{-\frac{2}{3}}, z=\frac{\text{Pi}}{4}, g(x,y,z)\right)$

$$\frac{3}{2}\cos(\pi)\,\pi^{1/3}\cos\left(\frac{1}{4}\pi\right)+\frac{1+\tan\left(\frac{1}{4}\pi\right)^2}{\tan\left(\frac{1}{4}\pi\right)} \tag{6}$$

> $simplify(\%)$

$$-\frac{3}{4}\pi^{1/3}\sqrt{2}+2 \tag{7}$$

b)

> $F := VectorField\left(\left\langle \frac{x^3}{\ln(x)},\ \mathrm{x}\cdot\mathrm{y}\cdot\ln(\mathrm{x}),\ z\cdot\ln(x\cdot y\cdot z)\right\rangle\right)$

$$F := \left(\frac{x^3}{\ln(x)}\right)\bar{e}_x+\left(x\,y\ln(x)\right)\bar{e}_y+\left(z\ln(x\,y\,z)\right)\bar{e}_z \tag{8}$$

> $g := (x,y,z)\rightarrow Divergence(F)$

$$g := (x,y,z)\rightarrow VectorCalculus\text{:-}Divergence(F) \tag{9}$$

> $g(x,y,z)$

$$\frac{3\,x^2}{\ln(x)}-\frac{x^2}{\ln(x)^2}+x\ln(x)+\ln(x\,y\,z)+1 \tag{10}$$

> $subs(x=\exp(1), y=\exp(1), z=\exp(3), g(x,y,z))$

$$\frac{3\,(\mathrm{e})^2}{\ln(\mathrm{e})}-\frac{(\mathrm{e})^2}{\ln(\mathrm{e})^2}+\mathrm{e}\ln(\mathrm{e})+\ln\left((\mathrm{e})^2\,\mathrm{e}^3\right)+1 \tag{11}$$

> $simplify(\%)$

$$2\,e^2 + e + 6 \tag{12}$$

>