Oppgave 4.6.6

b)

Med skrittlengde $h = 0.5$ må n gå fra $n = 1$ til $n = \frac{2}{0.5} = 4$ i **for**-løkken

Ved Eulers metode er $c(n) = c(n-1) + 10^{-8} \cdot \left(10^6 - \mathrm{c}(\mathrm{n}-1)\right) \cdot \left(10^5 - c(n-1)\right) \cdot h$.

> $c(0) := 0$

$$c(0) := 0 \tag{1}$$

> **for** n **from** 1 **by** 1 **to** 4 **do** $t := c(n-1)$: $c(n) := \mathit{evalf}\left(\mathrm{t} + 10^{-8} \cdot \left(10^6 - \mathrm{t}\right) \cdot \left(10^5 - \mathrm{t}\right) \cdot 0.5\right)$ **end do**

$$t := 0$$
$$c(1) := 500.0000000$$
$$t := 500.0000000$$
$$c(2) := 997.2512500$$
$$t := 997.2512500$$
$$c(3) := 1491.771341$$
$$t := 1491.771341$$
$$c(4) := 1983.577726 \tag{2}$$

(På spørsmål fra Maple svarer du at det snakk om Table assignment.) Legg merke til at kolon brukes til å skille mellom operasjonene som Maple skal gjennomføre i hvert trinn av løkken, og at man ikke må trykke linjeskift før man er ferdig med å skrive inn hele **for**-løkke-kommandoen. Maple sørger selv for linjeskift om nødvendig.

c)

Med skrittlengde $h = 0.5$ må n gå fra $n = 1$ til $n = \frac{2}{0.5} = 4$ i **for**-løkken

Ved Eulers midtpunktmetode er $c(n) = c(n-1) + 10^{-8} \cdot \left(10^6 - \mathrm{cc}(\mathrm{n}-1)\right) \cdot \left(10^5 - cc(n-1)\right) \cdot h$

der $cc(n-1) = c(n-1) + 10^{-8} \cdot \left(10^6 - c(n-1)\right)\left(10^5 - c(n-1)\right) \cdot \left(\frac{h}{2}\right)$.

> $c(0) := 0$

(3

$$c(0) := 0 \tag{3}$$

> **for** n **from** 1 **by** 1 **to** 4 **do** $t := c(n-1)$: $q := t + 10^{-8}\cdot(10^6 - t)\cdot(10^5 - t)\cdot\left(\frac{0.5}{2}\right)$: $c(n) := t + 10^{-8}\cdot(10^6 - q)\cdot(10^5 - q)\cdot 0.5$ **end do**

$$t := 0$$
$$q := 250.0000000$$
$$c := n \to t + 0.5\left(\frac{1}{100} - \frac{1}{100000000}q\right)(100000 - q)$$
$$t := 498.6253125$$
$$q := 747.2547145$$
$$c := n \to t + 0.5\left(\frac{1}{100} - \frac{1}{100000000}q\right)(100000 - q)$$
$$t := 994.5182035$$
$$q := 1241.785751$$
$$c := n \to t + 0.5\left(\frac{1}{100} - \frac{1}{100000000}q\right)(100000 - q)$$
$$t := 1487.696092$$
$$q := 1733.610461$$
$$c := n \to t + 0.5\left(\frac{1}{100} - \frac{1}{100000000}q\right)(100000 - q) \tag{4}$$

d)
Med skrittlengde $h = 1$ må n gå fra $n = 1$ til $n = 2$ i **for**-løkken.
Ved Runge-Kuttas metode av fjerde orden er

$$c(n) = c(n-1) + \frac{h}{6}\left(m1(n-1) + 2\cdot m2(n-1) + 2\cdot m3(n-1) + m4(n-1)\right)$$

der $m1(n-1) = 10^{-8}\cdot\left(10^6 - c(n-1)\right)\cdot\left(10^5 - c(n-1)\right)$

$$m2(n-1) = 10^{-8}\cdot\left(10^6 - c(n-1) - m1(n-1)\cdot\left(\frac{h}{2}\right)\right)\cdot\left(10^5 - c(n-1) - m1(n-1)\cdot\left(\frac{h}{2}\right)\right)$$

$$m3(n-1) = 10^{-8}\cdot\left(10^6 - c(n-1) - m2(n-1)\cdot\left(\frac{h}{2}\right)\right)\cdot\left(10^5 - c(n-1) - m2(n-1)\cdot\left(\frac{h}{2}\right)\right)$$

$$m4(n-1) = 10^{-8}\cdot\left(10^6 - c(n-1) - m3(n-1)\cdot \mathrm{h}\right)\cdot\left(10^5 - c(n-1) - m3(n-1)\cdot \mathrm{h}\right)$$

> $c(0) := 0$

$$c(0) := 0 \qquad (5$$

> **for** n **from** 1 **by** 1 **to** 2 **do** $t := c(n-1) : \quad m1 := 10^{-8}\cdot(10^6 - t)\cdot(10^5 - t) : \quad m2 := 10^{-8}\cdot\left(10^6 - t - \frac{m1}{2}\right)\cdot\left(10^5 - t - \frac{m1}{2}\right) : \quad m3 := 10^{-8}\cdot\left(10^6 - t - \frac{m2}{2}\right)\cdot\left(10^5 - t - \frac{m2}{2}\right) : \quad m4 := 10^{-8}\cdot(10^6 - t - m3)\cdot(10^5 - t - \mathrm{m3}) : \quad c(n) := t + \frac{1}{6}\cdot(m1 + 2\cdot m2 + 2\cdot m3 + m4)$ **end do**

Oj! Siden vi har skrevet inn at $h = 1$ istedenfor $h = 1.0$, har vi fått svarene ut som brøker. Det var ikke lurt. Vi skifter til desimaltall:

$$t := 0$$

$$m1 := 1000$$

$$m2 := \frac{397801}{400}$$

$$m3 := \frac{63650093365635601}{640000000000000}$$

$$m4 := \frac{40512308476144470781874921117871863120 1}{40960000000000000000000000000000000}$$

$$c(1) := \frac{24441407278415804006187492111787186312 01}{24576000000000000000000000000000000}$$

$$t := \frac{2444140727841580400618749211178718631201}{2457600000000000000000000000000000000}$$

$$m1 := \frac{597378357545879679178577400911932286875338518545626855466665925177813960359050702401}{603979776000000000000000000000000000000000000000000000000000000000000000000000000}$$

$m2 :=$

14352980102342574183944609031477842023120751388882178490568916829147627220355659775600925869503090982542\
7745708602577272661753711159158554418524294826448324042814651467164801 /
14591662792680407040000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000

$m3 :=$

83776073791903196594552389961625136191920587771837222001799420730348726057390730296007818158080766051839\
81297680208815013709153342445141314724694617004034045510347895742656489936328727743248246620686284048796\
76249743784969935669145797566703715262581811849027269332670498833542838898046135748987301483373016628319\
5102937549064016741845046841853293369601 /
85166649222117510176971466040032624640000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000

$m4 :=$

70958078009384473556301427841915836781896960447905801807275788230794266716991439008036843829755448863423\
15808583853228458380102449919301148244719000541363331644867467570523766994798051331126588262711001372839\
422451451311132080314047623219775844015611568115166251078576967589286163903594619155073592382510488189495\
41415111028752452879723660467389917337617209397777565725904819942158152019168953234442860944062758790909\
86367132196314716129977959702571179034002378214357210097214266695952747568309576328388180742964466102744\
69887358938061803820581294873639993613608770197604853933252222791034295235369406464795999624431176828 15\
4824857579932913903892029838041431616732955809040744101753995265250535487028790899201 /
72533581397232091013283017897107797695693763199613133423396555155671351296000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\

00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000000000

$c(2) :=$ **(6**

86090972527174689057173280207528765089331958437139884922885892224950161560968948799620436940008915203733\
95977337944264709368536093178356055167736882995522438589933733695754463834446625307863926825657551018434\
43938402517831692348781947395284677970318823245217554865788494711213193344520731474203064619796010739869\
81139200948354120070906999746007277973052836851831553358980464803796567593289695323444286094406275879090\
98636713219631471612997795970257117903400237821435721009721426669595274756830957632838818074296446610274\
4698873589380618038205812948736399936136087701976048539332522227910342952353694064647959996244311768281\
54824857579932913903892029838041431616732955809040744101753995265250535487028790899201 /
43520148838339254607969810738264678617416257919767880054037933093402810777600000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
00000000000000000000000000000000000000000000000000000000000000000000000000000000000

> **for** n **from** 1 **by** 1 **to** 2 **do** $t := c(n-1):\ \ m1 := 10^{-8}\cdot(10^{6}-t)\cdot(10^{5}-t):\ \ m2 := 10^{-8}\cdot(10^{6}-t-m1\cdot 0.5)\cdot(10^{5}-t-m1\cdot 0.5):\ \ m3 := 10^{-8}\cdot(10^{6}-t-m2\cdot 0.5)\cdot(10^{5}-t-m2\cdot 0.5):\ \ m4 := 10^{-8}\cdot(10^{6}-t-m3)\cdot(10^{5}-t-\mathrm{m3}):\ \ c(n) := t+\frac{1}{6}\cdot(m1+2\cdot m2+2\cdot m3+m4)$ **end do**

$$t := 0$$

$$m1 := 1000$$

$$m2 := 994.5025000$$

$$m3 := 994.5327089$$

$$m4 := 989.0700312$$
$$c(1) := 994.5234082$$
$$t := 994.5234082$$
$$m1 := 989.0701333$$
$$m2 := 983.6425297$$
$$m3 := 983.6723008$$
$$m4 := 978.2789798$$
$$c(2) := 1978.186537 \quad \textbf{(7}$$

Oppgave 4.6.7

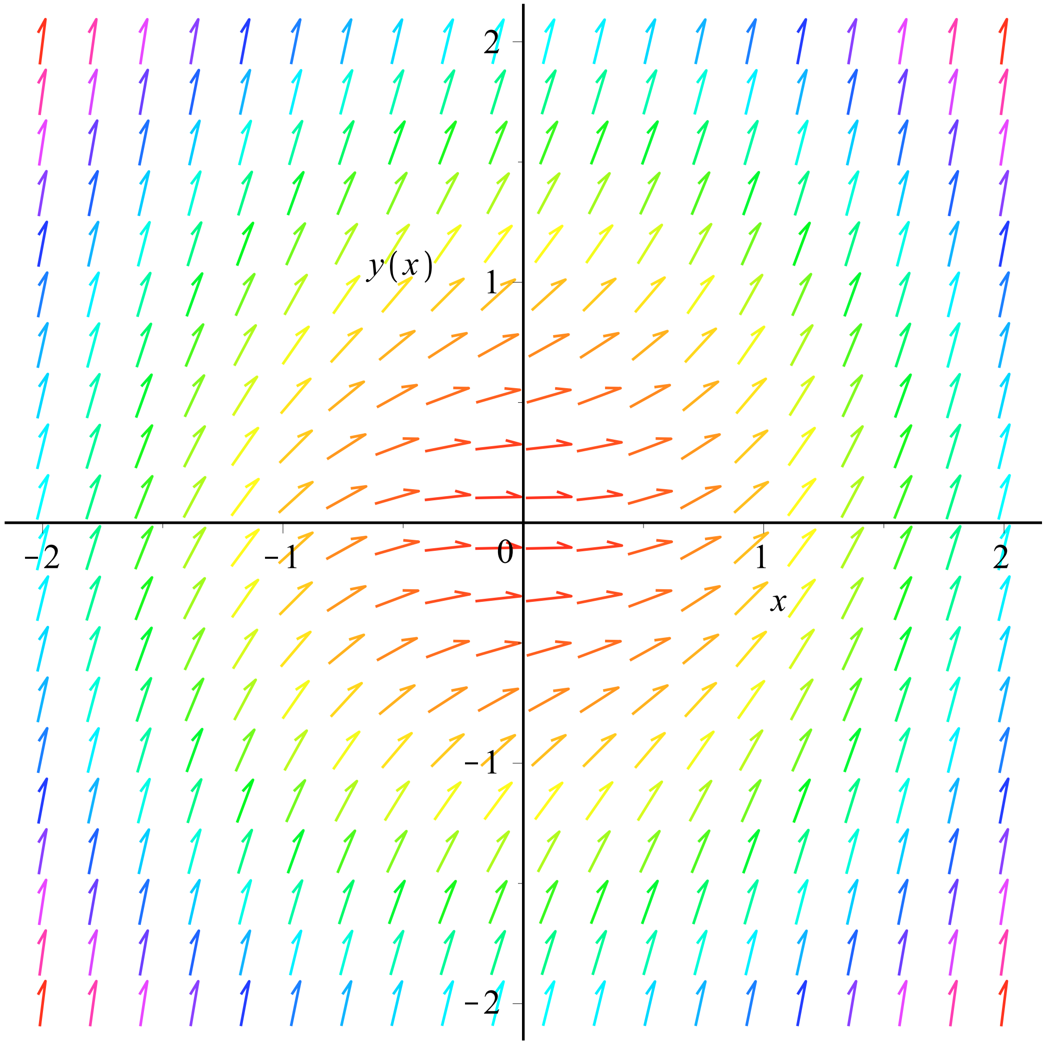
Retningsfelt er svært tidkrevende å tegne for hånd, men Maple gjør jbben i en fei.
Men først må Maple hente inn spesielle kommandoer for differensiallikninger:

> with(DEtools)

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, **(8** *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*]

Kommandoen for å tegne retningsfelt er *dfieldplot*
For å fortelle hvilken differensiallikning Maple skal finne retningsfeltet for, må vi naturligvis skrive inn den.
Husk da at $\mathit{diff}(y(x), x)$ betyr den deriverte av y med hensyn på x
Vi må også fortelle hvilken størrelse som er den ukjente i likningen, hvilken variabel som er den ukjente funksjonen, og variasjonsområdene for x og y som vi vil ha med i plottet.
Vi kan også trikse litt med farger i plottet hvis vi vil.

> $\mathit{dfieldplot}\left(\mathit{diff}(y(x), x) = x^2 + y(x)^2, y(x), x = -2..2, y = -2..2, \mathit{color} = x^2 + y^2\right)$



Legg merke til at vi skriver $y(x)$ for y i differensiallikningen.
color = *tall* er en måte å angi farge på. Siden $x^2 + y^2$ varierer fra punkt til punkt i feltet, varierer fargen tilsvarende.
Maple har laget alle pilene like lange, slik at man lettere ser retningen til feltet.

b)

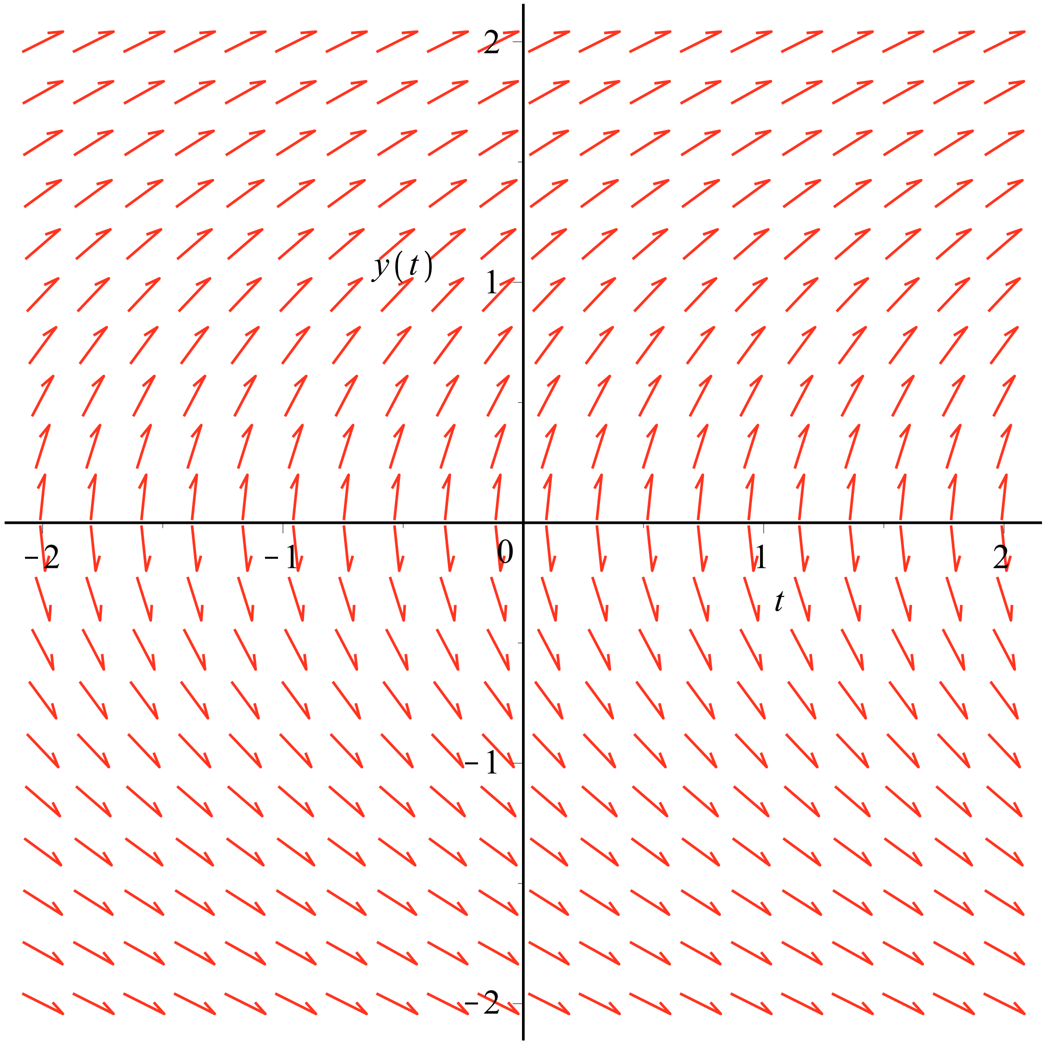
> $dfieldplot\left(diff(y(t), t) = \frac{1}{y(t)}, y(t), t=-2..2, y=-2..2\right)$

Error, invalid input: diff received 994.5234082, which is not valid for its 2nd argument

Æsjda: Feilmelding. Det ser ut som om noen av de variable allerede ligger som tall hos Maple. Vi tar en liten unassign først:

> $unassign('y', 't')$

> $dfieldplot\left(diff(y(t), t) = \frac{1}{y(t)}, y(t), t=-2..2, y=-2..2\right)$



Oppgavene 4.6.8 og 4.6.9 overlates til egentrening.