Ekstraoppgave 8.6.1.

a)

(i)

Vi henter først inn Maplekommandoer for vektorregning:

> $with(VectorCalculus)$

$[\&x,$ `*`, `+`, `-`, `.`, $<,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, \mathrm{D}, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series]$ **(1**

Det enkleste er å definere posisjonsvektoren som en vektorvaluert funksjon:

> $r := t \rightarrow \langle \cos(t), \sin(t) \rangle$

$$r := t \rightarrow VectorCalculus:\text{-}<,>(\cos(t), \sin(t)) \qquad \mathbf{(2}$$

> $v := t \rightarrow diff(r(t), t)$

$$v := t \rightarrow \frac{\mathrm{d}}{\mathrm{d}t} r(t) \qquad \mathbf{(3}$$

> $v(t)$

$$-\sin(t) e_x + (\cos(t)) e_y \qquad \mathbf{(4}$$

Legg merke til notasjonen her. Maple bruker betegnelsen e_x for enhetsvektoren $\langle 1, 0 \rangle$ (eller <1,0,0> i $\mathbb{R}^3$) og e_y for $\langle 0, 1 \rangle$ (eller $\langle 0, 1, 0 \rangle$ i $\mathbb{R}^3$). (Naturligvis betyr da også $e_z = \langle 0, 0, 1 \rangle$.)

> $a := t \rightarrow diff(v(t), t)$

$$a := t \to \frac{\mathrm{d}}{\mathrm{d}t}\, v(t) \qquad (5$$

> $a(t)$

$$-\cos(t)e_x - \sin(t)e_y \qquad (6$$

(ii)

For å dekomponere akselerasjonsvektoren $a(\pi)$, trenger vi å finne både $a(\pi)$ og enhetstangentvektoren $\boldsymbol{T} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ i punktet der $t = \pi$. Maple bruker kommandoen $Norm(v(t))$ for lengden $|\boldsymbol{v}(t)|$ av en vektor

> $T := t \to \frac{v(t)}{Norm(v(t))}$

$$T := t \to v(t)\, \frac{1}{VectorCalculus{:}\text{-}Norm(v(t))} \qquad (7$$

> $T(t)$

$$-\sin(t)e_x + (\cos(t))e_y \qquad (8$$

Vi er så klare til å sette inn at $t = \mathrm{Pi}$: (Husk, Maple oppfatter aPi som en ny størrelse.)

> $aPi := subs(t = \mathrm{Pi}, a(t))$

$$aPi := -\cos(\pi)e_x - \sin(\pi)e_y \qquad (9$$

> aPi

$$e_x \qquad (10$$

> $TPi := subs(t = \mathrm{Pi}, T(t))$

(11

$$TPi := -\sin(\pi)e_x + (\cos(\pi))e_y \tag{11}$$

> TPi

$$-e_y \tag{12}$$

Nå ser vi at dekomponeringen ble spesielt enkel i dette eksemplet, for $a(\pi) \perp T(\pi)$. Derved er tangensialkomponenten av $a(\pi)$ lik 0, og normalkomponenten er $a(\pi) = e_x = \langle 1, 0 \rangle$.
La oss likevel la Maple regne det ut, for å demonstrere hvordan det kan gjøres også i mer kompliserte tilfeller:

Tangensialkomponenten til akselerasjonen i punktet er

> $aT := DotProduct(aPi, TPi) \cdot TPi$

$$aT := 0e_x \tag{13}$$

altså, ingen tangensialkomponent. Hele skaelerasjonen er derfor normal til bevegelsen (hvilket vi naturligvis visste hele tiden, for dette er en partikel som går med konstant fart rundt på en sirkel).
Normalkomponenten er altså

> $\mathrm{aN} := aPi - \mathrm{aT}$

$$aN := e_x \tag{14}$$

b)

(i)

> $r := t \rightarrow \left\langle 2 \cdot \cos(2\,t), 4\sin(2\,t), \frac{t}{4} \right\rangle$

$$r := t \rightarrow VectorCalculus{:-}{<,>}\left(2\cos(2\,t), 4\sin(2\,t), 1\,\frac{1}{4}\,t\right) \tag{15}$$

> $v := t \rightarrow diff(r(t), t)$

$$v := t \to \frac{d}{dt}\, r(t) \tag{16}$$

> $v(t)$

$$-4\sin(2\,t)e_x + 8\cos(2\,t)e_y + \left(\frac{1}{4}\right)e_z \tag{17}$$

> $a := t \to diff(v(t), t)$

$$a := t \to \frac{d}{dt}\, v(t) \tag{18}$$

> $a(t)$

$$-8\cos(2\,t)e_x - 16\sin(2\,t)e_y \tag{19}$$

(ii)

> $T := t \to \frac{v(t)}{Norm(v(t))}$

$$T := t \to v(t)\, \frac{1}{VectorCalculus{:}\text{-}Norm(v(t))} \tag{20}$$

> $T(t)$

$$-\frac{16\sin(2\,t)}{\sqrt{768\cos(2\,t)^2 + 257}}e_x + \frac{32\cos(2\,t)}{\sqrt{768\cos(2\,t)^2 + 257}}e_y + \left(\frac{1}{\sqrt{768\cos(2\,t)^2 + 257}}\right)e_z \tag{21}$$

> $T0 := \mathrm{subs}(\mathrm{t} = 0, \mathrm{T(t)})$

$$T0 := -\frac{16\sin(0)}{\sqrt{768\cos(0)^2 + 257}}e_x + \frac{32\cos(0)}{\sqrt{768\cos(0)^2 + 257}}e_y + \left(\frac{1}{\sqrt{768\cos(0)^2 + 257}}\right)e_z \tag{22}$$

> T0

$$\frac{32}{1025}\sqrt{1025}\, e_y + \frac{1}{1025}\sqrt{1025}\, e_z \tag{23}$$

> $a0 := \mathrm{subs}(\mathrm{t} = 0, \mathrm{a(t)})$

(24

$$a0 := -8\cos(0)e_x - 16\sin(0)e_y \quad (24$$

> $a0$

$$-8e_x \quad (25$$

Tangensialkomponenten av akselerasjonen er derved

> $aT := DotProduct(a0, T0) \cdot T0$

$$aT := 0e_x \quad (26$$

altså er den null, mens normalvektoren er

> $aN := a0 - aT$

$$aN := -8e_x \quad (27$$

Ekstraoppgave 8.6.2.

a)

(i)

> $r := t \rightarrow \langle \exp(t), t^2, t \rangle$

$$r := t \rightarrow VectorCalculus{:}\text{-}{<,>}\left(e^t, t^2, t\right) \quad (28$$

Følgende tre kommandoer gir oss vektorer med riktig retning:

> $TT := TangentVector(r(t))$

(29

$$TT := \begin{bmatrix} e^t \\ 2\,t \\ 1 \end{bmatrix} \tag{29}$$

> $NN := PrincipalNormal(r(t))$

$$NN := \begin{bmatrix} \frac{e^t \left(4\,t^2 - 4\,t + 1\right)}{\left(1 + e^{2\,t} + 4\,t^2\right)^{3/2}} \\ -\frac{2\left(t\,e^{2\,t} - e^{2\,t} - 1\right)}{\left(1 + e^{2\,t} + 4\,t^2\right)^{3/2}} \\ -\frac{e^{2\,t} + 4\,t}{\left(1 + e^{2\,t} + 4\,t^2\right)^{3/2}} \end{bmatrix} \tag{30}$$

> $BB := Binormal(r(t))$

$$BB := \begin{bmatrix} -\frac{2}{1 + e^{2\,t} + 4\,t^2} \\ \frac{e^t}{1 + e^{2\,t} + 4\,t^2} \\ -\frac{2\,e^t\,(t-1)}{1 + e^{2\,t} + 4\,t^2} \end{bmatrix} \tag{31}$$

Men dette er ikke enhetsvektorer. For å finne enhetsvektorene, må vi dividere på lengden (normen) av vektoren:

> $T := \frac{TT}{Norm(TT)}$

(32

$$T := \begin{bmatrix} \dfrac{e^t}{\sqrt{1 + (e^t)^2 + 4t^2}} \\ \dfrac{2t}{\sqrt{1 + (e^t)^2 + 4t^2}} \\ \dfrac{1}{\sqrt{1 + (e^t)^2 + 4t^2}} \end{bmatrix} \tag{32}$$

> $N := \dfrac{NN}{Norm(NN)}$

$$N := \begin{bmatrix} \dfrac{e^t (4t^2 - 4t + 1)}{\sqrt{\dfrac{4 e^{2t} t^2 - 8 t e^{2t} + 5 e^{2t} + 4}{(1 + e^{2t} + 4t^2)^2}}\,(1 + e^{2t} + 4t^2)^{3/2}} \\ -\dfrac{2 (t e^{2t} - e^{2t} - 1)}{\sqrt{\dfrac{4 e^{2t} t^2 - 8 t e^{2t} + 5 e^{2t} + 4}{(1 + e^{2t} + 4t^2)^2}}\,(1 + e^{2t} + 4t^2)^{3/2}} \\ -\dfrac{e^{2t} + 4t}{\sqrt{\dfrac{4 e^{2t} t^2 - 8 t e^{2t} + 5 e^{2t} + 4}{(1 + e^{2t} + 4t^2)^2}}\,(1 + e^{2t} + 4t^2)^{3/2}} \end{bmatrix} \tag{33}$$

> $B := \dfrac{BB}{Norm(BB)}$

$$B := \left[\begin{array}{c} -\frac{2}{\sqrt{\frac{4\,e^{2\,t}\,t^2 - 8\,t\,e^{2\,t} + 5\,e^{2\,t} + 4}{\left(1 + e^{2\,t} + 4\,t^2\right)^2}}\,\left(1 + e^{2\,t} + 4\,t^2\right)} \\ \frac{e^t}{\sqrt{\frac{4\,e^{2\,t}\,t^2 - 8\,t\,e^{2\,t} + 5\,e^{2\,t} + 4}{\left(1 + e^{2\,t} + 4\,t^2\right)^2}}\,\left(1 + e^{2\,t} + 4\,t^2\right)} \\ -\frac{2\,e^t\,(t-1)}{\sqrt{\frac{4\,e^{2\,t}\,t^2 - 8\,t\,e^{2\,t} + 5\,e^{2\,t} + 4}{\left(1 + e^{2\,t} + 4\,t^2\right)^2}}\,\left(1 + e^{2\,t} + 4\,t^2\right)} \end{array}\right] \tag{34}$$

Egentlig kan vi få alle disse tre vektorene ved rett og slett spørre etter:

> $TNBFrame(r(t))$

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$$\left[\begin{array}{c}\dfrac{e^{t}}{\sqrt{1+e^{2t}+4t^{2}}}\\[2ex]\dfrac{2t}{\sqrt{1+e^{2t}+4t^{2}}}\\[2ex]\dfrac{1}{\sqrt{1+e^{2t}+4t^{2}}}\end{array}\right], \left[\begin{array}{c}\dfrac{e^{t}\left(4t^{2}-4t+1\right)}{\sqrt{\dfrac{4e^{2t}t^{2}-8te^{2t}+5e^{2t}+4}{\left(1+e^{2t}+4t^{2}\right)^{2}}}\left(1+e^{2t}+4t^{2}\right)^{3/2}}\\[2ex]-\dfrac{2\left(te^{2t}-e^{2t}-1\right)}{\sqrt{\dfrac{4e^{2t}t^{2}-8te^{2t}+5e^{2t}+4}{\left(1+e^{2t}+4t^{2}\right)^{2}}}\left(1+e^{2t}+4t^{2}\right)^{3/2}}\\[2ex]-\dfrac{e^{2t}+4t}{\sqrt{\dfrac{4e^{2t}t^{2}-8te^{2t}+5e^{2t}+4}{\left(1+e^{2t}+4t^{2}\right)^{2}}}\left(1+e^{2t}+4t^{2}\right)^{3/2}}\end{array}\right], \tag{35}$$

$$\left[\begin{array}{c}-\dfrac{2}{\sqrt{\dfrac{4e^{2t}t^{2}-8te^{2t}+5e^{2t}+4}{\left(1+e^{2t}+4t^{2}\right)^{2}}}\left(1+e^{2t}+4t^{2}\right)}\\[2ex]\dfrac{e^{t}}{\sqrt{\dfrac{4e^{2t}t^{2}-8te^{2t}+5e^{2t}+4}{\left(1+e^{2t}+4t^{2}\right)^{2}}}\left(1+e^{2t}+4t^{2}\right)}\\[2ex]-\dfrac{2e^{t}\left(t-1\right)}{\sqrt{\dfrac{4e^{2t}t^{2}-8te^{2t}+5e^{2t}+4}{\left(1+e^{2t}+4t^{2}\right)^{2}}}\left(1+e^{2t}+4t^{2}\right)}\end{array}\right]$$

(ii)

> $C := Curvature(r(t))$

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$$C := \frac{1}{2}\,\frac{1}{\sqrt{1+e^{2t}+4t^2}}\left(4\left(-\frac{1}{2}\,\frac{e^t\left(2e^{2t}+8t\right)}{\left(1+e^{2t}+4t^2\right)^{3/2}}+\frac{e^t}{\sqrt{1+e^{2t}+4t^2}}\right)^2+4\left(-\frac{t\left(2e^{2t}+8t\right)}{\left(1+e^{2t}+4t^2\right)^{3/2}}+\frac{2}{\sqrt{1+e^{2t}+4t^2}}\right)^2+\frac{\left(2e^{2t}+8t\right)^2}{\left(1+e^{2t}+4t^2\right)^3}\right)^{1/2} \quad (36$$

> $To := Torsion(r(t))$

$$To := -\left(2\left(-\frac{1}{2}\left(e^t\left(4t^2-4t+1\right)\left(\frac{2\left(e^t\right)^2\left(4t^2-4t+1\right)^2}{\left(1+e^{2t}+4t^2\right)^3}+\frac{2\left(e^t\right)^2\left(4t^2-4t+1\right)\left(8t-4\right)}{\left(1+e^{2t}+4t^2\right)^3}\right.\right.\right.\right.$$

$$-\frac{3\left(e^t\right)^2\left(4t^2-4t+1\right)^2\left(2e^{2t}+8t\right)}{\left(1+e^{2t}+4t^2\right)^4}+\frac{8\left(te^{2t}-e^{2t}-1\right)\left(-e^{2t}+2te^{2t}\right)}{\left(1+e^{2t}+4t^2\right)^3}$$

$$\left.\left.-\frac{12\left(te^{2t}-e^{2t}-1\right)^2\left(2e^{2t}+8t\right)}{\left(1+e^{2t}+4t^2\right)^4}+\frac{2\left(e^{2t}+4t\right)\left(2e^{2t}+4\right)}{\left(1+e^{2t}+4t^2\right)^3}-\frac{3\left(e^{2t}+4t\right)^2\left(2e^{2t}+8t\right)}{\left(1+e^{2t}+4t^2\right)^4}\right)\right)\Bigg/$$

$$\left(\left(\frac{\left(e^t\right)^2\left(4t^2-4t+1\right)^2}{\left(1+e^{2t}+4t^2\right)^3}+\frac{4\left(te^{2t}-e^{2t}-1\right)^2}{\left(1+e^{2t}+4t^2\right)^3}+\frac{\left(e^{2t}+4t\right)^2}{\left(1+e^{2t}+4t^2\right)^3}\right)^{3/2}\left(1+e^{2t}+4t^2\right)^{3/2}\right)$$

$$+\frac{e^{t}\left(4t^{2}-4t+1\right)}{\sqrt{\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(t\,e^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}}\,\left(1+e^{2t}+4t^{2}\right)^{3/2}}$$

$$+\frac{e^{t}\left(8t-4\right)}{\sqrt{\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(t\,e^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}}\,\left(1+e^{2t}+4t^{2}\right)^{3/2}}$$

$$\left.\left.-\frac{3}{2}\,\frac{e^{t}\left(4t^{2}-4t+1\right)\left(2e^{2t}+8t\right)}{\sqrt{\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(t\,e^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}}\,\left(1+e^{2t}+4t^{2}\right)^{5/2}}\right)\right)\Bigg/$$

$$\left(\sqrt{\left(\frac{4}{\left(1+e^{2t}+4t^{2}\right)^{2}}+\frac{\left(e^{t}\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{2}}+\frac{4\left(e^{t}\right)^{2}\left(t-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{2}}\right)\left(1+\left(e^{t}\right)^{2}+4t^{2}\right)}\,\left(1+e^{2t}+4t^{2}\right)\right)+\left(\left(\left(\left(t\,e^{2t}-e^{2}\right.\right.\right.\right.$$

$$\left.\left.+\frac{2\left(e^{2t}+4t\right)\left(2e^{2t}+4\right)}{\left(1+e^{2t}+4t^{2}\right)^{3}}-\frac{3\left(e^{2t}+4t\right)^{2}\left(2e^{2t}+8t\right)}{\left(1+e^{2t}+4t^{2}\right)^{4}}\right)\right)\Bigg/\left(\left(\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}\right.\right.$$

$$\left.\left.+\frac{4\left(t\,e^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}\right)^{3/2}\left(1+e^{2t}+4t^{2}\right)^{3/2}\right)$$

$$-\frac{2\left(-e^{2t}+2te^{2t}\right)}{\sqrt{\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(te^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}}\;\left(1+e^{2t}+4t^{2}\right)^{3/2}}$$

$$\left.\left.+\frac{3\left(te^{2t}-e^{2t}-1\right)\left(2e^{2t}+8t\right)}{\sqrt{\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(te^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}}\;\left(1+e^{2t}+4t^{2}\right)^{5/2}}\right)e^{t}\right)\Bigg/$$

$$\left(\sqrt{\left(\frac{4}{\left(1+e^{2t}+4t^{2}\right)^{2}}+\frac{\left(e^{t}\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{2}}+\frac{4\left(e^{t}\right)^{2}\left(t-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{2}}\right)\left(1+\left(e^{t}\right)^{2}+4t^{2}\right)}\;\left(1+e^{2t}+4t^{2}\right)\right)$$

$$-\left(2\left(\frac{1}{2}\,\frac{\left(e^{2t}+4t\right)\left(\frac{2\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{2\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)\left(8t-4\right)}{\left(1+e^{2t}+4t^{2}\right)^{3}}-\frac{3\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}\left(2e^{2t}+\cdots\right.}{\left(1+e^{2t}+4t^{2}\right)^{4}}\cdots\right.}{\left(\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(\cdots\right.}{\left(\cdots\right.}\cdots\right.}\right.\right.$$

$$-\frac{2e^{2t}+4}{\sqrt{\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(te^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}}\;\left(1+e^{2t}+4t^{2}\right)^{3/2}}$$

$$\left.\left.+\frac{3}{2}\,\frac{\left(e^{2t}+4t\right)\left(2e^{2t}+8t\right)}{\sqrt{\frac{\left(e^{t}\right)^{2}\left(4t^{2}-4t+1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{4\left(te^{2t}-e^{2t}-1\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}+\frac{\left(e^{2t}+4t\right)^{2}}{\left(1+e^{2t}+4t^{2}\right)^{3}}}\;\left(1+e^{2t}+4t^{2}\right)^{5/2}}\right)e^{t}\left(t-1\right)\right)$$

$$\Bigg/ \left(\sqrt{\left(\frac{4}{\left(1+e^{2t}+4t^2\right)^2} + \frac{\left(e^t\right)^2}{\left(1+e^{2t}+4t^2\right)^2} + \frac{4\left(e^t\right)^2 (t-1)^2}{\left(1+e^{2t}+4t^2\right)^2} \right) \left(1+\left(e^t\right)^2+4t^2\right)} \left(1+e^{2t}+4t^2\right) \right)$$

Dette ble voldsomme greier!!!
Vi prøver *simplify* :

> $To := simplify(\%)$

$$To := -\frac{2\,e^t}{\left(1+e^{2t}+4t^2\right)^{3/2} \sqrt{\frac{4e^{2t}t^2-8te^{2t}+5e^{2t}+4}{\left(1+e^{2t}+4t^2\right)^2}} \sqrt{\frac{4e^{2t}t^2-8te^{2t}+5e^{2t}+4}{1+e^{2t}+4t^2}}} \quad (38)$$

Det ble heldigvis bedre!!

(iii)

> subs(t = 0, T)

$$\begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 0 \\ \frac{1}{2}\sqrt{2} \end{bmatrix} \quad (39)$$

> subs(t = 0, N)

$$\begin{bmatrix} \frac{1}{36}\sqrt{9}\sqrt{4}\sqrt{2} \\ \frac{1}{9}\sqrt{9}\sqrt{4}\sqrt{2} \\ -\frac{1}{36}\sqrt{9}\sqrt{4}\sqrt{2} \end{bmatrix} \quad (40$$

> *simplify*(%)

$$\begin{bmatrix} \frac{1}{6}\sqrt{2} \\ \frac{2}{3}\sqrt{2} \\ -\frac{1}{6}\sqrt{2} \end{bmatrix} \quad (41$$

> subs(t = 0, B)

$$\begin{bmatrix} -\frac{1}{9}\sqrt{9}\sqrt{4} \\ \frac{1}{18}\sqrt{9}\sqrt{4} \\ \frac{1}{9}\sqrt{9}\sqrt{4} \end{bmatrix} \quad (42$$

> *simplify*(%)

$$\begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \quad (43$$

> subs(t = 0, C)

$$\frac{1}{2}\frac{\sqrt{4\left(-\frac{(e^0)^2}{(1+e^0)^{3/2}}+\frac{e^0}{\sqrt{1+e^0}}\right)^2+\frac{16}{1+e^0}+\frac{4\,(e^0)^2}{(1+e^0)^3}}}{\sqrt{1+e^0}} \quad (44$$

> *simplify*(%)

$$\frac{3}{4}\sqrt{2} \quad (45$$

> *subs*(t = 0, To)

$$-\frac{2\,e^0}{(1+e^0)^{3/2}\sqrt{\frac{4+5\,e^0}{(1+e^0)^2}}\sqrt{\frac{4+5\,e^0}{1+e^0}}} \quad (46$$

> simplify(%)

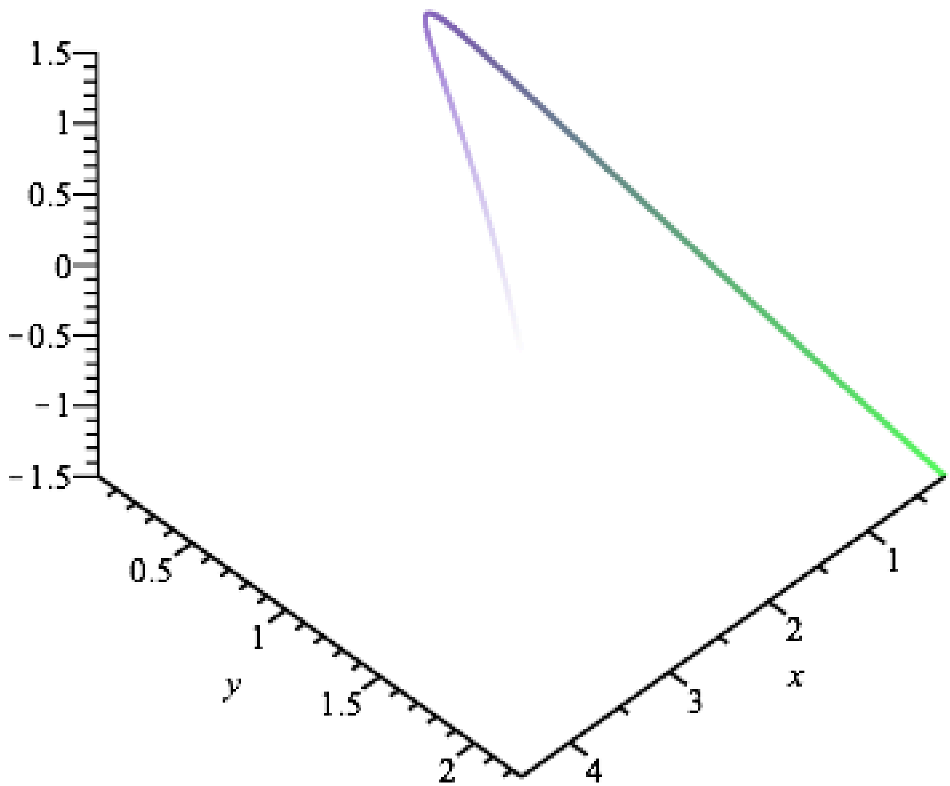
$$-\frac{2}{9} \quad (47$$

For å plotte romkurven, trenger vi å hente inn Maples plottekommandoer:

> *with*(*plots*)

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] (48

> *spacecurve*([exp(*t*), t^2, *t*, *t* = -1.5 ..1.5], *axes* = *framed*, *labels* = [*x*, *y*, *z*])


1.5
1
0.5
0
-0.5
-1
-1.5
0.5
1
y
1.5
2
1
2
x
3
4

>