Oppgave 10.7.18.

Vi starter med å hente inn Maples vektorregningskommandoer:

```
> with(VectorCalculus)
```

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series] **(1**

Det enkleste vi kan gjøre, er å erstatte punktet $\left(\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}\right)$ med punktet $\left(\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}\right) + 0.1 \cdot \frac{\nabla F\left(\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}\right)}{\left|\nabla F\left(\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}\right)\right|}$.

Derfor prøver vi det. Vi definerer først funksjonen:

$$> F := (x, y, z) \to \frac{z \cdot \cot\left((\sin(x \cdot y))^2\right)}{\mathrm{sqrt}\left(x^3 + y^3\right)} + y \cdot \cos(z)^2$$

$$F := (x, y, z) \to z \cot\left(\sin(x\,y)^2\right) \frac{1}{\sqrt{x^3 + y^3}} + y \cos(z)^2 \qquad \textbf{(2}$$

For å beregne ∇F skal vi bruke kommandoen *Gradient*, men det krever at vi henter inn Maples kommandoer for VectorCalculus

```
> with(VectorCalculus)
```

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, **(3**

IsVectorField, *Jacobian*, *Laplacian*, *LineInt*, *MapToBasis*, *Nabla*, *Norm*, *Normalize*, *PathInt*, *PlotPositionVector*, *PlotVector*, *PositionVector*, *PrincipalNormal*, *RadiusOfCurvature*, *RootedVector*, *ScalarPotential*, *SetCoordinateParameters*, *SetCoordinates*, *SpaceCurve*, *SurfaceInt*, *TNBFrame*, *Tangent*, *TangentLine*, *TangentPlane*, *TangentVector*, *Torsion*, *Vector*, *VectorField*, *VectorPotential*, *VectorSpace*, *Wronskian*, *diff*, *eval*, *evalVF*, *int*, *limit*, *series*]

> $G := Gradient(F(x, y, z), [x, y, z])$

$$G := \left(\frac{2 z \left(-1-\cot\left(\sin(x y)^2\right)^2\right) \sin(x y) \cos(x y)\, y}{\sqrt{x^3+y^3}} - \frac{3}{2}\, \frac{z \cot\left(\sin(x y)^2\right) x^2}{\left(x^3+y^3\right)^{3/2}}\right) \bar{e}_x + \left(\frac{2 z \left(-1-\cot\left(\sin(x y)^2\right)^2\right) \sin(x y) \cos(x y)\, x}{\sqrt{x^3+y^3}} - \frac{3}{2}\, \frac{z \cot\left(\sin(x y)^2\right) y^2}{\left(x^3+y^3\right)^{3/2}} + \cos(z)^2\right) \bar{e}_y + \left(\frac{\cot\left(\sin(x y)^2\right)}{\sqrt{x^3+y^3}} - 2 y \cos(z) \sin(z)\right) \bar{e}_z \tag{4}$$

> $GG := subs\left(\mathrm{x} = \frac{\mathrm{Pi}}{4}, y = \frac{\mathrm{Pi}}{3}, z = \frac{\mathrm{Pi}}{6}, G\right)$

$$GG := \left(\frac{1}{819}\, \frac{\pi^2 \left(-1-\cot\left(\sin\left(\frac{1}{12}\pi^2\right)^2\right)^2\right) \sin\left(\frac{1}{12}\pi^2\right) \cos\left(\frac{1}{12}\pi^2\right) \sqrt{91}\, \sqrt{1728}}{\sqrt{\pi^3}} - \frac{27}{8281}\, \frac{\pi^3 \cot\left(\sin\left(\frac{1}{12}\pi^2\right)^2\right) \sqrt{91}\, \sqrt{1728}}{\left(\pi^3\right)^{3/2}}\right) \bar{e}_x + \left(\frac{1}{1092}\, \frac{\pi^2 \left(-1-\cot\left(\sin\left(\frac{1}{12}\pi^2\right)^2\right)^2\right) \sin\left(\frac{1}{12}\pi^2\right) \cos\left(\frac{1}{12}\pi^2\right) \sqrt{91}\, \sqrt{1728}}{\sqrt{\pi^3}} \tag{5}$$

$$-\frac{48}{8281}\frac{\pi^3\cot\left(\sin\left(\frac{1}{12}\pi^2\right)^2\right)\sqrt{91}\sqrt{1728}}{\left(\pi^3\right)^{3/2}}+\frac{3}{4}\Bigg)\bar{e}_y+\left(\frac{1}{91}\frac{\cot\left(\sin\left(\frac{1}{12}\pi^2\right)^2\right)\sqrt{91}\sqrt{1728}}{\sqrt{\pi^3}}-\frac{1}{6}\pi\sqrt{3}\right)\bar{e}_z$$

Svaret vi fikk, ser voldsomt ut. Men det er ikke behov for det eksakte svaret

> $GGf := evalf(GG)$

$$GGf := (-2.024959302)\bar{e}_x + (-1.169525781)\bar{e}_y + (0.4074561273)\bar{e}_z \qquad \textbf{(6}$$

Dette er altså gradienten til $F(x, y, z)$ i det gitte punktet. Vi vil ha enhetsvektoren i denne retningen. Det får vi til med kommandoen *Normalize* :

> $GGn := Normalize(GGf)$

$$GGn := (-0.853095054728077)\bar{e}_x + (-0.492709487624108)\bar{e}_y + (0.171657181887542)\bar{e}_z \qquad \textbf{(7}$$

Nå virker det ganske kunstig å tviholde på det opprinnelige punktet på eksakt form. Derfor ber vi også om å få det på desimalform:

> $P := evalf\left(\left\langle \frac{\text{Pi}}{4}, \frac{\text{Pi}}{3}, \frac{\text{Pi}}{6} \right\rangle\right)$

$$P := (0.7853981635)e_x + (1.047197551)e_y + (0.5235987758)e_z \qquad \textbf{(8}$$

Nå er det fristende å skrive

> $Pny := P + 0.1 \cdot GGn$

```
Error, (in VectorCalculus:-+) cannot add a free Vector and a vector field
```

Men det gikk ikke. Maple oppfatter *Gradient(...)* som et vektorfelt som den behandler på en annen måte enn vanlige vektorer. (Vektorfelter er noe vi kommer tilbake til i kapittel 12.)
Dette viser Maple ved at det settes små streker over enhetsvektorene e_x, e_y og e_z.

Derfor må vi skrive punktet P som et vektorfelt også:

> $P := VectorField\left(\left\langle \frac{\mathrm{Pi}}{4}, \frac{\mathrm{Pi}}{3}, \frac{\mathrm{Pi}}{6} \right\rangle, \text{'}cartesian\text{'}[x, y, z]\right)$

$$P := \frac{1}{4}\pi\bar{e}_x + \frac{1}{3}\pi\bar{e}_y + \frac{1}{6}\pi\bar{e}_z \qquad \textbf{(9}$$

> $Pny := P + 0.1 \cdot GGn$

$$Pny := \left(\frac{1}{4}\pi - 0.0853095054728077\right)\bar{e}_x + \left(\frac{1}{3}\pi - 0.0492709487624108\right)\bar{e}_y + \left(\frac{1}{6}\pi + 0.0171657181887542\right)\bar{e}_z \qquad \textbf{(10}$$

> $evalf(Pny)$

$$(0.700088658027192)\bar{e}_x + (0.997926602237589)\bar{e}_y + (0.540764493988754)\bar{e}_z \qquad \textbf{(11}$$

For å kontrollere om det nye punktet virkelig gir en forbedring, sjekker vi funksjonsverdien i det gamle punktet P og sammenligner med funksjonsverdien i det nye punktet Pny:

> $evalf\left(F\left(\frac{\mathrm{Pi}}{4}, \frac{\mathrm{Pi}}{3}, \frac{\mathrm{Pi}}{6}\right)\right)$

$$1.473593256 \qquad \textbf{(12}$$

> $evalf(F(0.700088658027192,\ 0.997926602237589,\ 0.540764493988754))$

$$1.798796949 \qquad \textbf{(13}$$

SUKSESS!

>